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A COMPUTERIZED ALGORITHM FOR THE
DETERMINATION OF THE OPTIMAL ALLOCATION
OF TWO WEAPONS SYSTEMS AGAINST N TARGETS

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Monterey, California

THESIS

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James Frederick Amerault

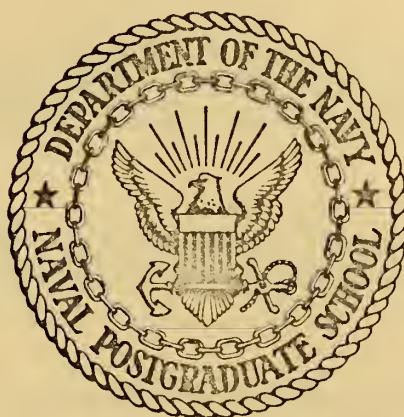Thesis Advisor: J. G. Taylor

September 1972

A Computerized Algorithm for the
Determination of the Optimal Allocation
of Two Weapons Systems Against N Targets

by

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ABSTRACT

The problem of allocating munitions from M weapons systems
to N target complexes is studied and a review of pertinent
literature is presented. An algorithm for the solution of
the problem in the special case of two weapons systems
against N targets is developed and programmed for computerized
solution. The results of an example problem are shown and
tested. Discussion of the algorithm's extension to more
than two weapons systems is included as are alternative
solution techniques.

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I.   INTRODUCTION

Defense planners are vitally concerned with the problem
of assigning weapons systems to strategic targets. Informa-
tion from intelligence sources is obtained, from which decisions
are made regarding the military value of such targets. Judge-
ments as to the effectiveness of weapons systems when used
against target types can be made on the basis of test data
gathered during Manufacturers' or Armed Services' observed
evaluation firings. Constraints on the numbers of units of
particular systems are dictated by Defense Department budget
or resource availability considerations.

All of the above factors are known to be subject to change
and can only be estimated to a varying degree of accuracy at
any given time. Because of the uncertain nature of these
factors, an optimum targeting policy often calls for a mix-
ture of weapons to be assigned to any single target complex.
Problems of this nature may be formulated as nonlinear-program-
ming problems.

The problem of optimal allocation of units of two weapons
systems against N targets or a complex of N targets is the
subject of this thesis. An algorithm for solution of this
problem is presented and a program is given for rapid solution
by computer.

II. STUDY OF PERTINENT LITERATURE

The problem considered by this paper is concerned with
the allocation of X and Y munitions of two particular weapons
systems among N target complexes. Koopman [1] pointed out
that the problem of optimum distribution of search effort
and the allocation of munitions projected against area targets
are analogous. Charnes and Cooper [4] provided a link between
search theory and linear programming. F. A. Miercort and
R. M. Soland [11] show an integer nonlinear programming
formulation of the problem of allocation from a single weapons
system against area and point defense. They develop a branch
and bound algorithm for its solution.

John Danskin formulated a nonlinear model of M weapons
systems' munitions assignment to N targets which he labelled
a "simple maximum" problem. The iterative method of solution
which he developed is the basis for the algorithm presented
in this paper [2].

Lemus and David [8] use an analytic technique similiar
to Danskin's to solve the problem and suggest that this should
be considered a general solution in the case where the attacker
has more than one type of weapon available for assignment to
an undefended or virtually undefended target complex. They
point out that whenever the number of weapons is large com-
pared with the number of targets that analytic solution methods
offer a great savings in computer time for little loss of
accuracy. Earlier writings dealing with solutions to problems

of this type, such as those by Manne [6] and by DenBroeder,
Ellison and Emerling [7], reformulate the allocation problem
as a linear program. The resulting linear program will
ordinarily provide a close enough approximation to the original
problem to achieve satisfactory results.

III. DEVELOPMENT OF THE SOLUTION ALGORITHM

A. CHARACTERIZATION OF THE OPTIMAL ALLOCATION

The allocation of rounds of two distinct weapons systems among N targets is a nonlinear programming problem, stated mathematically as

$$\text{Maximize} \quad \sum_{i=1}^{N} V_i \left\{ 1 - \exp\left(-\left[\mu_i x_i + \nu_i y_i \right] \right) \right\} \quad ,$$

$$\text{Subject to:} \quad \sum_{i=1}^{N} x_i \leq X$$

$$\sum_{i=1}^{N} y_i \leq Y$$

$$x_i, y_i \geq 0 \text{ for all } i \quad ,$$

where V_i represents the military worth of target i, μ_i is the effectiveness of system X against target i and v_i is the effectiveness of system Y against target i. The linear constraints imply that the well-known Kuhn-Tucker constraint qualification holds at all points belonging to the constraint region. Since the objective function is concave as well, our problem becomes one of concave programming and the denoted Kuhn-Tucker conditions are necessary and sufficient for an optimal allocation. Let us denote the optimal allocation as

$$x^* = (x_1^*, x_2^*, \cdot \cdot \cdot, x_N^*) \quad , \quad y^* = (y_1^*, y_2^*, \cdot \cdot \cdot, y_N^*)$$

The Lagrangian function is given by

$$L(x, y, \lambda) = \sum_{i=1}^{N} V_i \left\{ 1 - \exp\left(-\left[\mu_i x_i + \nu_i y_i \right] \right) \right\} + \lambda_1 \left(X - \sum_{i=1}^{N} x_i \right) + \lambda_2 \left(Y - \sum_{i=1}^{N} y_i \right),$$

such that

$$\frac{\partial L}{\partial x_i} = \mu_i V_i \exp\left(-[\mu_i x_i + \tau_i y_i]\right) - \lambda_1$$

and

$$\frac{\partial L}{\partial y_i} = \tau_i V_i \exp\left(-[\mu_i x_i + \tau_i y_i]\right) - \lambda_2 \ .$$

For the above problem, the necessary and sufficient Kuhn-Tucker conditions are;

1) $$\frac{\partial L}{\partial x_i} \ , \ \frac{\partial L}{\partial y_i} \leq 0$$

2) $$x_i^* \frac{\partial L}{\partial x_i} = 0 \ , \ y_i^* \frac{\partial L}{\partial y_i} = 0$$

3) $$x_i^* \ , \ y_i^* \geq 0$$

4) $$\lambda_1^* \ , \ \lambda_2^* \geq 0$$

5) $$\lambda_1^* \left(X - \sum_{i=1}^{N} x_i^*\right) = 0 \ , \ \lambda_2^* \left(Y - \sum_{i=1}^{N} y_i^*\right) = 0$$

6) $$X - \sum_{i=1}^{N} x_i^* \leq 0 \quad , \quad Y - \sum_{i=1}^{N} y_i^* \leq 0$$

Consideration of $\frac{\partial L}{\partial x_i} \leq 0$ implies

$$\mu_i V_i \exp\left(-[\mu_i x_i^* + \tau_i y_i^*]\right) \leq \lambda_1^* \ .$$

$x_i^* \frac{\partial L}{\partial x_i} = 0$ implies;

$x_i^* = 0$ if and only if $\mu_i V_i \exp\left(-[\tau_i y_i^*]\right) \leq \lambda_1^*$

$x_i^* > 0$ whenever $\mu_i V_i \exp\left(-[\mu_i x_i^* + \tau_i y_i^*]\right) = \lambda_1^*$.

It is easily shown that $\lambda_i^* > 0$. For example, consider λ_1^*, which we know by 4) satisfied

$$\lambda_1^* \geq 0 ,$$

if $\lambda_1^* = 0$, then $\mu_i V_i > 0 = \lambda_1^*$ for all i.

Hence

$$x_i^* = \lim_{\lambda_1 \to 0} \left\{ \frac{1}{\mu_i} \ln \left[\frac{\mu_i x_i^* + \nu_i y_i^*}{\lambda_1^*} \right] \right\} = +\infty$$

and the constraint is violated.

In like manner, $y_i^* = 0$ if and only if

$$\nu_i V_i \exp\left(-[\mu_i x_i^*]\right) \leq \lambda_2^* ,$$

$y_i^* > 0$ whenever $\nu_i V_i \exp\left(-[\mu_i x_i^* + \nu_i y_i^*]\right) = \lambda_2^*$

and $\lambda_2^* > 0$.

The above considerations yield that there are four possibilities for optimal allocation against a target;

1) $x_i^* = 0$, $y_i^* = 0$

then
$$\mu_i V_i \leq \lambda_1^* , \quad \nu_i V_i \leq \lambda_2^*$$

2) $x_i^* > 0$, $y_i^* = 0$

then $\mu_i V_i \exp\left(-\mu_i x_i^*\right) = \lambda_1^*$,

$$\nu_i V_i \exp\left(-\mu_i x_i^*\right) \leq \lambda_2^*$$

and $x_i^* = \frac{1}{\mu_i} \ln\left(\frac{\mu_i V_i}{\lambda_1^*}\right)$

3) $y_i^* > 0$, $x_i^* = 0$

then, similiarly,

$$y_i^* = \frac{1}{\nu_i} \ln\left(\frac{\nu_i V_i}{\lambda_2^*}\right)$$

4) The case of a shared target, $x_i^*, y_i^* > 0$.

then $\quad \mu_i V_i \exp(-[\mu_i x_i^* + v_i y_i^*]) = \lambda_1^*$,

$\qquad v_i V_i \exp(-[\mu_i x_i^* + v_i y_i^*]) = \lambda_2^*$

and $\quad \dfrac{\mu_i}{v_i} = \dfrac{\lambda_1^*}{\lambda_2^*}$ $\qquad$ so that;

$$x_i^* = \dfrac{\ln.\left(\dfrac{\mu_i V_i}{\lambda_1^*}\right) - v_i y_i^*}{\mu_i}$$

$$y_i^* = \dfrac{\ln.\left(\dfrac{v_i V_i}{\lambda_2^*}\right) - \mu_i x_i^*}{v_i} .$$

This paper assumes that for all i and j

$$\dfrac{\mu_i}{v_i} \neq \dfrac{\mu_j}{v_j} ,$$

and thus there can exist at most one shared target. Two cases are then possible for analysis, that of one shared target and that of no shared targets.

Case 1): No shared targets;

If no shared targets exists, either x_i^* or $y_i^* = 0$, such that $x_i^* = 0$ whenever $y_i^* > 0$ and $y_i^* = 0$ whenever $x_i^* > 0$.

Letting $\quad x_i^* > 0$,

then $\quad x_i^* = \dfrac{1}{\mu_i} \ln.\left(\dfrac{\mu_i V_i}{\lambda_1^*}\right)$

and $\quad \displaystyle\sum_{\forall x_i^* > 0} x_i^* = X \;\Rightarrow\; \sum_{\forall x_i^* > 0} x_i^* = \sum_{\forall x_i^* > 0} \dfrac{1}{\mu_i} \ln(\lambda_1^*)$

or, $\quad \lambda_1^* = \exp\left\{ \dfrac{\left[\displaystyle\sum_{\forall x_i^* > 0} \dfrac{1}{\mu_i} \ln(\mu_i V_i)\right] - X}{\displaystyle\sum_{\forall x_i^* > 0} \dfrac{1}{\mu_i}} \right\}$

and likewise,

$\quad \lambda_2^* = \exp\left\{ \dfrac{\left[\displaystyle\sum_{\forall y_i^* > 0} \dfrac{1}{v_i} \ln(v_i V_i)\right] - Y}{\displaystyle\sum_{\forall y_i^* > 0} \dfrac{1}{v_i}} \right\} .$

Case 2):   One shared target;

If target j is shared, then

$$\frac{\mu_j}{\nu_j} = \frac{\lambda_1^*}{\lambda_2^*} \Rightarrow \lambda_2^* = \frac{\lambda_1^* \, \nu_j}{\mu_j} \quad . \text{(a)}$$

It is known that,

$$\sum_{\substack{\forall x_i^* > 0 \\ i \neq j}} \frac{1}{\mu_i} \ln\left(\frac{\mu_i V_i}{\lambda_1^*}\right) + x_j^* = X$$

and therefore,

$$x_j^* = X - \sum_{\substack{\forall x_i^* > 0 \\ i \neq j}} \frac{1}{\mu_i} \ln\left(\frac{\mu_i V_i}{\lambda_1^*}\right) . \quad \text{(b)}$$

Likewise,

$$y_j^* = Y - \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} \frac{1}{\nu_i} \ln\left(\frac{\nu_i V_i}{\lambda_2^*}\right) .$$

Substituting from (a) then,

$$y_j^* = Y - \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} \frac{1}{\nu_i} \ln\left(\frac{\nu_i V_i \, \mu_j}{\lambda_1^* \, \nu_j}\right) . \quad \text{(c)}$$

From allocation rule 4), the shared target case;

$$\mu_j x_j^* + \nu_j y_j^* = \ln\left(\frac{\mu_j V_j}{\lambda_1^*}\right) = \ln\left(\frac{\nu_j V_j}{\lambda_2^*}\right) . \quad \text{(d)}$$

Substituting (b) and (c) in (d) leads to the result:

$$Z = \lambda_1^* = \exp\left\{ \frac{\left[\sum_{\substack{\forall x_i^* > 0}} \mu_j/\mu_i \ln\left(\mu_i V_i\right) + \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} \nu_j/\nu_i \ln\left(\frac{\mu_j V_i}{\nu_j/\nu_i}\right) - \mu_j - \nu_j \right]}{\left[1 + \sum_{\substack{\forall x_i^* > 0 \\ i \neq j}} \mu_j/\mu_i + \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} \nu_j/\nu_i \right]} \right\}$$

and from

$$\lambda_2^* = \lambda_1^*\left(\frac{\nu_j}{\mu_j}\right) ,$$

then

$$\lambda_2^* = Z\left(\frac{\nu_j}{\mu_j}\right) .$$

B. AN ALGORITHM FOR COMPUTER SOLUTION

The non-linear program solved in part A. is identical to
the "simple maximum" problem studied by John Danskin [2].
Danskin developed an iterative procedure for solution of this
problem. the ideas of this algorithm were employed to outline
a step by step solution suitable for computerization. Program-
ming and testing this procedure forms the basis of this report.
The program, as is the case in Danskin's original algorithm,
yields allocations of rounds of weapons systems one and two
for each target. Figure (1) presents a graphical flow-chart
of this algorithm.

C. COMPUTER PROGRAM FOR SOLUTION:

Computer Program A solves the Weapons Systems Allocation
problem for two systems against N targets. The problem is
written in FORTRAN for solution on an IBM system 360 computer.
Computer Program A shows the data required to solve the example
problem of Section IV in the initialization statements. Data
required to solve any problem are as follows:

N = number of targets

X = rounds available from system 1

y = rounds available from system 2

Box MU (I) = effectiveness of system 1 versus target I

Box ETA (I) = effectiveness of system 2 versus target I

BOX VEE (I) = military value of target I

Box K (I) = target number of target I

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Have shared target

Compute λ_1^*, λ_2^* by Formula

Compute x_i^*, y_i^*

END

Start

Solve for X System only obtain $\hat{\lambda}_1$, $\hat{x}_i$ for $i = 1, \ldots, N$

Yes

$S_y(\hat{\lambda}_2) > Y$

No

Using $\lambda_2 = \lambda_1 \left(\frac{V_j}{\mu_j}\right)$, Compute $S_y(\hat{\lambda}_2)$

Compute $r_i V_i e^{-\mu_i \hat{x}_i}$
$= \begin{cases} r_i V_i & \text{if } \hat{x}_i = 0 \\ \frac{r_i}{\mu_i} \hat{\lambda}_1 & \text{if } \hat{x}_i > 0 \end{cases}$

Compute $r_i V_i e^{-\mu_i \hat{x}_i}$
$= \begin{cases} r_i V_i & \text{if } \hat{x}_i = 0 \\ \frac{r_i}{\mu_i} \hat{\lambda}_1 & \text{if } \hat{x}_i > 0 \end{cases}$

For Y System order Targets as $r_j V_j e^{-\mu_j \hat{x}_j}$ decreases

Resolve X problem using $\mu_i V_i e^{-r_i \hat{y}_i}$, get new $\hat{\lambda}_1$

$j = 1$

Let $\hat{\lambda}_2 = r_j V_j e^{-\mu_j \hat{x}_j}$

END

Delete Tgt. j from X list

Compute $\hat{y}_i$, then $S_y(\hat{\lambda}_2)$

Compute x_i^*, y_i^*

$j = j + 1$

Use current $\hat{\lambda}_2$ Compute $\mu_i V_i e^{-r_i \hat{y}_i}$
$= \begin{cases} \mu_i V_i & \text{if } \hat{y}_i = 0 \\ \frac{\mu_i}{r_i} \hat{\lambda}_2 & \text{if } \hat{y}_i > 0 \end{cases}$

Yes

$\hat{\lambda}_1 > \hat{\lambda}_2 \left(\frac{\mu_i}{r_i}\right)$

No

$S_y(\hat{\lambda}_2) > Y$

Yes

$\lambda_1^* = \hat{\lambda}_1$ Compute λ_2^*
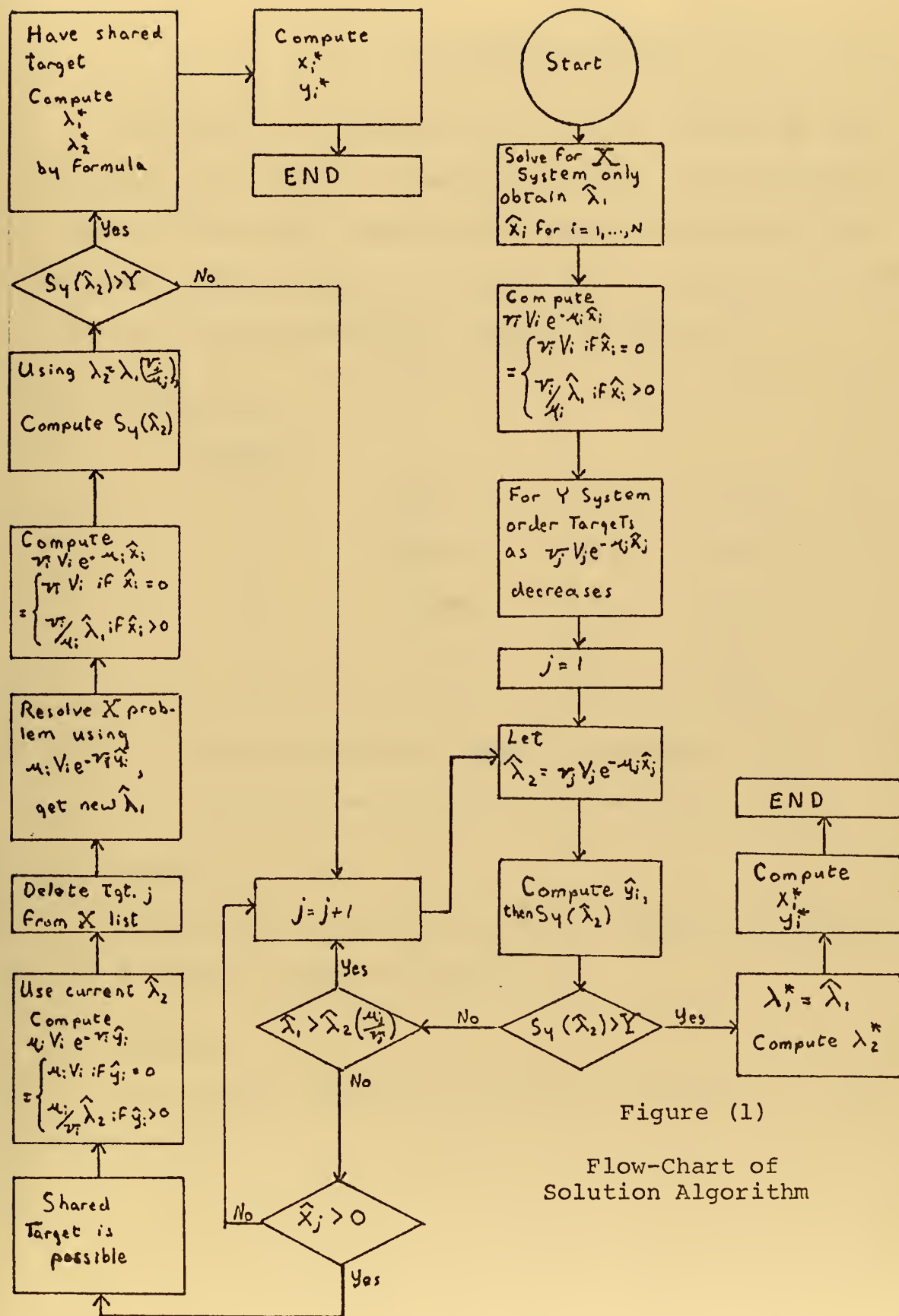
No

Shared Target is possible

No

$\hat{x}_j > 0$

Yes

Figure (1)

Flow-Chart of Solution Algorithm

IV. AN EXAMPLE PROBLEM

An example is presented here to show the use of the solution algorithm and computer program. It represents a sample of the data required for a problem involving two weapons systems against a complex of five targets and gives the results as computed by Computer Program A.

A. PROBLEM DATA

Target	V		
1	1000.0	0.03	0.005
2	100.0	0.20	0.20
3	500.0	0.02	0.06
4	25.0	0.20	2.00
5	10.0	0.20	0.20

Units of Weapons System 1 available: 80.0

Units of Weapons System 2 available: 50.0

B. PROBLEM

Compute the optimum allocation of rounds of systems 1 and 2 against targets 1 through 5.

C. RESULTS

The solution obtained from Computer Program A is as shown below.

C   COMPUTER OUTPUT   15

```
C       MUNITIONS AVAILABLE FROM WEAPONS SYSTEM 1 =80.0
        MUNITIONS AVAILABLE FROM WEAPONS SYSTEM 2 =50.0


C       OPTIMAL LAGRANGE MULTIPLIER FOR SYSTEM 2 =2.7925
C       OPTIMAL LAGRANGE MULTIPLIER FOR SYSTEM 1 =2.7925


C       OPTIMUM ALLOCATIONS FOR SYSTEM 1
C           TGT         ROUNDS
C            1          79.1423
C            2           0.8577
C            3           0.0000
C            4           0.0000
C            5           0.0000

C       OPTIMUM ALLOCATIONS FOR SYSTEM 2
C           TGT         ROUNDS
C            1           0.0000
C            2           8.9863
C            3          39.5711
C            4           1.4425
C            5           0.0000

C       SHARED TARGET IS TARGET 2
```

These results were satisfactorily checked against allocation rules 1-4 as outlined in Section III. The algorithm was also hand computed for this problem and results were compatible. The problem of integer solution was not addressed, and so results must be rounded off to the nearest whole integer for satisfactory use. Program running time for this example problem was 8.38 seconds.

V. EXTENSION OF DANSKIN'S ALGORITHM TO M WEAPONS SYSTEMS

The characterization and existence of a solution to the non-linear programming problem representing allocation of munitions from two weapons systems among N targets were shown in Section III. In Danskin's work he proved that if a solution existed for the M-1 weapons systems case, then one iteration of his algorithm from the M-1 base case would result in the proper allocation in the M systems case [2]. It is possible, though tedious, to extend the computerized solution algorithm one step at a time from the two weapons systems case to the M weapons systems case.

VI. OTHER SOLUTION APPROACHES

Lemus and David [10] state that, in the case where the number of weapons is comparable with the number of targets, the allocation problem may be reduced to a formulation of the transportation problem and solved by linear programming. This method has the advantage of integer-valued solution but is disadvantageous since the optimal allocation is characterized by (except for alternate optima) all fire being concentrated on as many targets as there are constraints.

In his paper, R. H. Day [9] shows a method of solution which decomposes the general allocation problem into a set of targeting problems in the small and a targeting problem in the large. He solves the former problems using a Sequential Optimization Method and employs the results to obtain a solution to the targeting problem in the large by nonlinear programming. Mylander [12] discusses how the problem can be solved using the Sequential Unconstrained Minimization Technique, (SUMT).

Ury Passy [10] showed that the general assignment problem could be formulated as a geometric program and could be solved through its dual. He developed an algorithm for solution of weapons-assignment problems such as the one addressed in this thesis.

VII. CONCLUSIONS

The computerized solution of the weapons systems alloca-
tion problem in the special case of two weapons systems
against N targets is shown. Extension of the solution proce-
dure to the case of M systems is discussed but not attempted.
Based on a survey of the literature, there appear to be
several alternate means of solution which could possibly be
profitably investigated.

The techniques studied should be useful in solving prob-
lems other than those of weapons allocation. Such problems
as allocation of Research and Development funds and alloca-
tion of search effort are examples [15].

Computer Program A

This program determines the allocation of X munitions from system 1
and Y munitions from system 2 among N targets.

```
      DIMENSION BOXX(5),EXSTAP(5),STAOP(5),STAOR(5),WYSTAP(5),EXHAT(5)
      DIMENSION BOXI(5),BOXJ(5),BOXK(5),BOXMU(5),BOXVEE(5),BOXETA(5)
      DIMENSION BOXL(5),YOU(5),YOUR(5),ELBOX(5),VEE(5)
      DATA BOXK/1.0,2.0,3.0,4.0,5.0/,BOXMU/0.03,0.2,0.02,0.02,0.2/
      DATA BOXVEE/1350.0,100.0,500.0,25.0,10.0/,BOXETA/0.305,0.2,0.06,
     12.0,0.2/
      N=5
      X=30.0
      Y=50.0
      ENLAM=0.0
      J=1
      DO 300 I=1,N
      A=BOXMU(I)*BOXVEE(I)
      BOXI(I)=A
      B=BOXETA(I)*BOXVEE(I)
      BOXJ(I)=B
  300 CONTINUE
      M=N
      MA=N
      NA=N
      NB=N
      NG=N
      WAD=ENLAM
      ESWHY=ENLAM
      ROLAM=ENLAM
      WTM=ENLAM
      WART=ENLAM
      ESS=ENLAM
      BPAIN=ENLAM
      BRAWN=ENLAM
      TROUT=ENLAM
      TRIPE=ENLAM
      ROUND=ENLAM
      N=N-1
 1000 DO 3000 I=1,N
      IF (BOXI(I).LT.BOXI(I+1)) GO TO 2000
      GO TO 3000
 2000 P=BOXI(I)
      BOXI(I)=BOXI(I+1)
      BOXI(I+1)=P
```

```
      Q=BOXMU(I)
      BOXMU(I)=BOXMU(I+1)
      BOXMU(I+1)=Q
      R=BOXVEE(I)
      BOXVEE(I)=BOXVEE(I+1)
      BOXVEE(I+1)=R
      S=BOXETA(I)
      BOXETA(I)=BOXETA(I+1)
      BOXETA(I+1)=S
      T=BOXJ(I)
      BOXJ(I)=BOXJ(I+1)
      BOXJ(I+1)=T
 3000 CONTINUE
      N=N-1
      IF (N.GE.1) GO TO 1000
      CONTINUE
      DO 600 I=1,M
      DALLAM=BOXI(I)
      DO 600 K=1,M
      IF (BOXI(K).LT.DALLAM) GO TO 600
      ESSLAM=1.0/BOXMU(K)
      ELLAM=(BOXI(K)/DALLAM)
      EXLAM=ESSLAM*ALOG(ELLAM)
      ENLAM=ENLAM+EXLAM
  600 CONTINUE
      BOXX(I)=ENLAM
      ENLAM=0.0
      IF (BOXX(I).GT.X) GO TO 9000
 6000 CONTINUE
      IF (I.GE.M) GO TO 90
 9000 I=I-1
   90 WAA=BOXI(I)
      WAB=X-BOXX(I)
      DO 60 L=1,I
      WAC=1/BOXMU(L)
      WAD=WAD+WAC
   60 CONTINUE
      WAE=WAB/WAD
      WAF=EXP(-WAE)
      STALAM=WAA*WAF
      DO 30 L=1,I
      WBA=1/BOXMU(L)
      WBB=BOXI(L)/STALAM
      WBC=WBA*ALOG(WBB)
      EXSTAR(L)=WBC
   30 CONTINUE
      I=I+1
      DO 20 L=I,M
```

```
20    EXSTAR(L)=0.0
      CONTINUE
      DO 99 I=1,M
      IF(EXSTAR(I).GT.0.0) GO TO 10
      STAOP(I)=BOXVEE(I)*BOXETA(I)
      STAOR(I)=STAOP(I)
      GO TO 99
10    STA=BOXETA(I)/BOXMU(I)
      STAOP(I)=STA*STALAM
      STAOR(I)=STAOP(I)
99    CONTINUE
      M=M-1
96    DO 98 I=1,M
      IF(STAOP(I).LT.STAOP(I+1)) GO TO 97
      GO TO 98
97    U=STAOP(I)
      STAOP(I)=STAOP(I+1)
      STAOP(I+1)=U
98    CONTINUE
      M=M-1
68    IF(M.GE.1) GO TO 96
      CONTINUE
      TWOLAM=STAOP(J)
      DO 89 I=1,MA
89    IF(STAOP(J).EQ.STAOR(I)) GO TO 87
87    CONTINUE
      NR=I
      DO 86 I=1,MA
      RUN=1/BOXETA(I)
      ROW=(BOXJ(I)/TWOLAM)
      ESWHY=RUN* ALOG(ROW)
      ESWAY=ESWHY+ESWAY
      IF(ESWAY.GT.0.0) GO TO 81
      ESWAY=0.0
81    BOXL(I)=ESWAY
86    CONTINUE
      IF(ESWHY.GT.Y) GO TO 85
      ESWHY=0.0
      SAM=BOXMU(NR)/(BOXETA(NR))
      RAS=TWOLAM*SAM
      IF(RAS.LT.STALAM) GO TO 69
      IF(EXSTAR(NR).GT.0.0) GO TO 65
69    J=J+1
      IF(J.GT.MA) GO TO 85
      GO TO 68
65    DO 67 I=1,MA
      IF(BOXL(I).GT.0.0) GO TO 66
      SEAT =BOXMU(I)*BOXVEE(I)
```

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```
        YOU(I)=SEAT
        GO TO 67
   66   TEA=BOXMU(I)/BOXETA(I)
        SEAT=TEA*TWOLAM
        YOU(I)=SEAT
   67   CONTINUE
        BOXK(NR)=0.0
        DO 58 I=1,NA
        YOUR(I)=YOU(I)
   58   CONTINUE
        NA=NA-1
   56   DO 59 I=1,NA
        IF (YOU(I).LT.YOU(I+1)) GO TO 57
        GO TO 59
   57   S=YOU(I)
        YOU(I)=YOU(I+1)
        YOU(I+1)=S
   59   CONTINUE
        NA=NA-1
        IF (NA.GE.1) GO TO 56
        DO 49 I=1,NB
        IF (BOXK(I).EQ.0.0) GO TO 49
        DALLAM=YOU(I)
        DO 48 K=1,NB
        IF (BOXK(K).EQ.0.0) GO TO 48
        ESSLAM=(1.0/BOXMU(K))
        ELLAM=(BOXI(K)/DALLAM)
        EXLAM=ESSLAM*ALOG(ELLAM)
        ROLAM=ROLAM+EXLAM
   48   CONTINUE
        ELBOX(I)=ROLAM
        IF(ROLAM.GT.X) GO TO 47
        ROLAM=0.0
   49   CONTINUE
   47   I=I-1
        IF(BOXK(I).EQ.0.0) GO TO 47
        DO 43 K=1,NB
        IF (YOU(I).FQ.YOUR(K)) GO TO 42
   43   CONTINUE
   42   NC=K
        DO 45 I=1,NC
        IF(BOXK(I).EQ.0.0) GO TO 45
        WEM=1/BOXMU(I)
        WTM=WTM+WEM
        WUM=ALOG(BOXI(I))
        WAR =WFM*WUM
        WART=WART+WAR
   45   CONTINUE
```

```fortran
      WARN=WART-X
      WARP=WARN/WTM
      WUNLAM=EXP(WARP)
      WTM=0.0
      WART=0.0
      DO 44 I=1,NB
      IF(BOXK(I).EQ.0.0) GO TO 40
      WOP=1/BOXMU(I)
      WEP=BOXI(I)/WUNLAM
      WIP= WOP*ALOG(WEP)
      EXHAT(I)=WIP
      GO TO 44
   40 EXHAT(I)=0.0
   44 CONTINUE
      DO 41 I=1,NB
      IF(EXHAT(I).LE.0.0) GO TO 39
      GO TO 41
   39 EXHAT(I)=0.0
   41 CONTINUE
      NC=NC+1
      DO 29 I=NC,NB
      EXHAT(I)=0.0
   29 CONTINUE
      DO 33 I=1,NB
      IF(EXHAT(I).EQ.0.0) GO TO 37
      MARS=BOXETA(I)/BOXMU(I)
      VEEP=MARS*WUNLAM
      GO TO 36
   37 VEEP=BOXJ(I)
   36 VEE(I)=VEEP
   38 CONTINUE
      DO 26 I=1,NG
      IF(BOXK(I).EQ.0.0) GO TO 28
      SHIP=BOXETA(I)/BOXMU(I)
      HIP=WUNLAM*SHIP
      TWOLAM=TIP
      GO TO 26
   28 CONTINUE
      DO 27 I=1,NG
      WY=1/BOXETA(I)
      WYY=VEE(I)/TWOLAM
      WYE=ALOG(WYY)
      WYEY=WY*WYE
      IF(WYEY.LE.0.0) GO TO 22
      ESS=ESS+WYEY
      WYSTAR(I)=WYEY
      GO TO 27
   22 WYSTAR(I)=0.0
```

```
 27   CONTINUE
      IF(ESS.LE.Y) GO TO 25
      ESS=0.0
      GO TO 24

 25   P=NR
      BOXK(NR)=R
      GO TO 69

 24   CONTINUE
      DO 532 I=1,NG
      IF(BOXK(I).EQ.0.0) GO TO 533
      GO TO 532

533   ZED=BOXMU(I)*X
      ZEP=BOXETA(I)*Y
      NS=I
      ZEE=ZED+ZEP

532   CONTINUE
      DO 632 I=1,NG
      IF(EXHAT(I).GT.0.0) GO TO 631
      IF(I.EQ.NS)GO TO 631
      GO TO 632

631   BRA=ALOG(BOXI(I))
      BRAT=BOXMU(NS)/BOXMU(I)
      BRAN=BRA*BRAT
      BRAIN=BRAIN+BRAN

632   CONTINUE
      DO 432 I=1,NG
      IF(EXHAT(I).GT.0.0) GO TO 431
      GO TO 432

431   IF(I.EQ.NS) GO TO 432
      BRAT=BOXMU(NS)/BOXMU(I)
      BRAWN=BRAWN+BRAT

432   CONTINUE
      DO 732 I=1,NG
      IF(WYSTAR(I).GT.0.0)GO TO 731
      GO TO 732

731   IF(I.EQ.NS) GO TO 732
      TPA=BOXETA(NS)/BOXETA(I)
      TRAIN=BOXMU(NS)*BOXVEE(I)
      TRAT=TRAIN/TRA
      TROT=ALOG(TRAT)
      TRIP=TRA*TROT
      TROUT=TROUT+TRIP
      TRIPE=TRIPE+TRA

732   CONTINUE
      WIPE=BRAWN+TRIPE+1.0
      WORK=BRAIN+TROUT-ZEE
      WORN=WORK/WIPE
      WUNLAM=EXP(WORN)
```

```
      WARP=BOXETA(NS)/BOXMU(NS)
      TWOLAM=WUNLAM*WARP
      DO 832 I=1,NS
      IF(I.EQ.NS)GO TO 832
      IF(I.EQ.NS)GO TO 832
      IF(EXHAT(I).LE.0.0) GO TO 831
      RANCH=1/BOXMJ(I)/WUNLAM
      RANCT=BOXI(I)/WUNLAM
      RATT=ALOG(RANCH)
      RALPH=RAUNCH*RATT
      EXSTAR(I)=RALPH
      ROUND=ROUND+RALPH
      GO TO 832
  831 EXSTAR(I)=0.0
  832 CONTINUE
      ROTT=X-ROUND
      EXSTAR(NS)=ROTT
      ROUND=0.0
      DO 932 I=1,NS
      IF(I.EQ.NS) GO TO 932
      IF(WYSTAR(I).LE.0.0) GO TO 931
      RAUNCT=1/BOXETA(I)/TWOLAM
      RANCT=BOXJ(I)/TWOLAM
      RATT=ALOG(RANCH)
      RALPH=RAUNCH*RATT
      WYSTAR(I)=RALPH
      ROUND=ROUND+RALPH
      GO TO 932
  931 WYSTAR(I)=0.0
  932 CONTINUE
      ROOT=Y-ROUND
      WYSTAR(NS)=ROOT
      GO TO 82
   85 WUNLAM=STALAM
      DO 84 I=1,NG
      DOG=1/BOXMU(I)
      CAT=BOXI(I)/WUNLAM
      WOLF=ALOG(CAT)
      EXSTAP(I)=DOG*WOLF
      IF(EXSTAR(I).GT.0.0) GO TO 83
      EXSTAR(I)=0.0
      WYSTAR(I)=1.0
      GO TO 84
   83 WYSTAR(I)=0.0
   84 CONTINUE
      DO 79 I=1,NG
      IF(WYSTAR(I).LE.0.0) GO TO 79
      ROPE=1/BOXETA(I)
      VERB=BOXJ(I)
```

```
      ROUND=ROUND+ROPE
      RICK=ALOG(VERB)
      DD=ROPE*RICK
      BRAIN=BRAIN+DD
79    CONTINUE
      ROUT=BRAIN-Y
      CANDY=ROUT/ROMP
      TWOLAM=EXP(CANDY)
      DO 78 I=1,NG
      DOG=1/BOXETA(I)
      CAT=BOXJ(I)/TWOLAM
      WOLF=ALOG(CAT)
      WYSTAR(I)=DOG*WOLF
78    CONTINUE
82    CONTINUE
9700  FORMAT (1X,F12.4)
      WRITE (6,9700) X,Y,WUNLAM,TWOLAM,EXSTAR,WYSTAR
      STOP
      END

//
```

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13. ABSTRACT

The problem of allocating munitions from M weapons systems to N target complexes is studied and a review of pertinent literature is presented. An algorithm for the solution of the problem in the special case of two weapons systems against N targets is developed and programmed for computerized solution. The results of an example problem are shown and tested. Discussion of the algorithm's extension to more than two weapons systems is included as are alternative solution techniques.
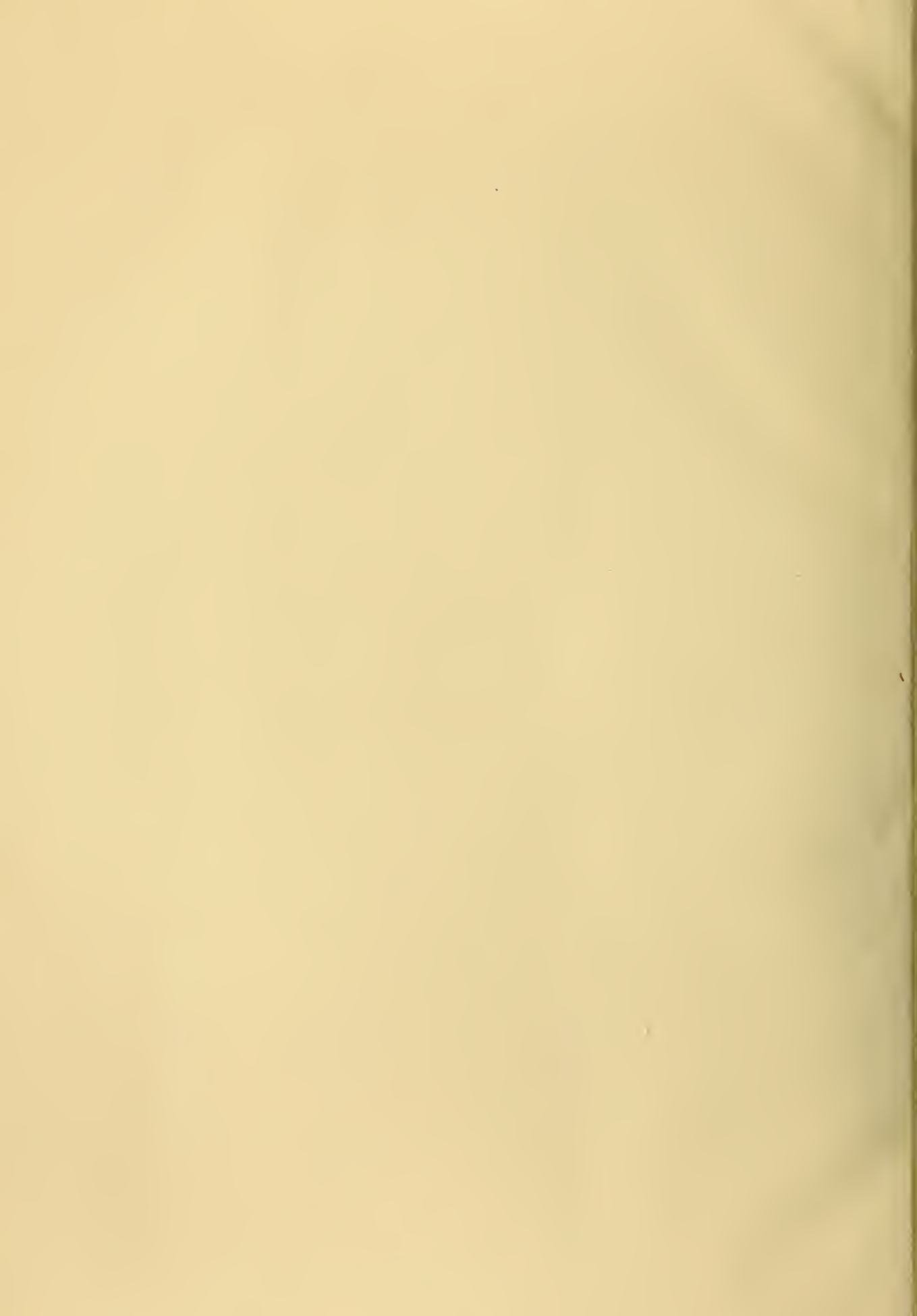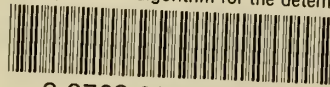
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